Short Course on Experimental Dynamic Substructuring

Module #07B: Payload and Component Simulations using Craig Bampton Form Modal Models (Effective Mass Models are Subset)

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Idealized models that help the engineer focus on the most important characteristics of the dynamics using fixed base modes

Short Course Notes For:
February 10, 2020, IMAC, Houston, TX, USA

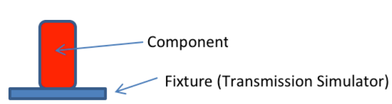
1

Craig Bampton Modal Model Concept

- Suppose we can define a component (or payload) model with the fixed base modes of the component and the modes of a fixture it is mounted upon
- By driving the fixture appropriately, we could then approximate component motion when mounted in a system due to the field inputs
- As a free structure, neglecting damping, assuming a rigid fixture for simplicity (not required), using p for fixed base modal dof and s for rigid fixture dof, the modal equation of motion will later be derived as:

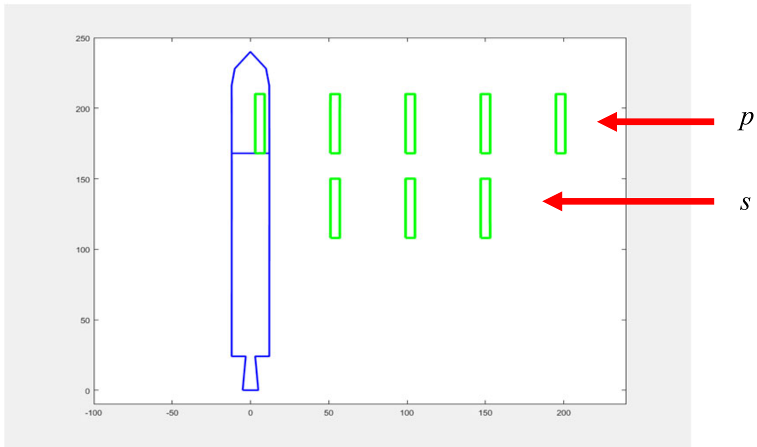
$$\left[\begin{bmatrix}\omega_{fix}^2 & 0\\ 0 & 0\end{bmatrix} - \omega^2\begin{bmatrix} I & M_{ps}\\ M_{ps}^T & M_{ss}\end{bmatrix}\right]\begin{Bmatrix}\bar{p}\\ \bar{s}\end{Bmatrix} = \begin{Bmatrix}0\\ \bar{F}_s\end{Bmatrix}$$

Matrices similar to Craig Bampton
Upper left matrices are diagonal



2

Craig Bampton modal model base driven to match rocket flight



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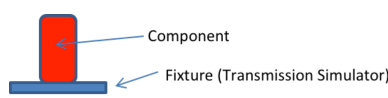
3

Theory (and practice) using the transmission simulator for Craig Bampton modal model

- Perform a free modal test, calculate fixture mass properties and component mass properties



$$\left[\omega^2{}_{free} + j2\omega\omega_{free}\zeta_{free} - \omega^2 I\right]\bar{q} = 0$$

$$\bar{q} = T\begin{Bmatrix}\bar{p}\\ \bar{s}\end{Bmatrix}$$

Φ – free mode shapes component and fixture

Ψ – free mode shapes of fixture

subscript b located on fixture

$$\Phi_b\bar{q} \approx \Psi\bar{s} \rightarrow \bar{q} = \Phi_b^+\Psi\bar{s}$$

apply TS fixed base constraint

$$\Psi^+\,\Phi_b\bar{q} = s = 0$$

$$\bar{q} = L_{fix}\Gamma\bar{p} \rightarrow T = \left[L_{fix}\Gamma \quad \Phi_b^+\Psi\right]$$

If Ψ is just rigid body modes, one only needs the rigid body Φ

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4

Resulting Craig Bampton modal model

$$\left[\begin{bmatrix}\omega_{fix}^2 & K_{ps}\\ K_{ps}^T & K_{ss}\end{bmatrix} - \omega^2\begin{bmatrix} I & M_{ps}\\ M_{ps}^T & M_{ss}\end{bmatrix}\right]\begin{Bmatrix}\bar{p}\\ \bar{s}\end{Bmatrix} = \begin{Bmatrix}0\\ \bar{F}_s\end{Bmatrix}$$

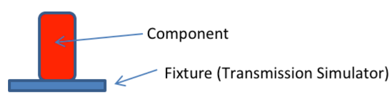
- With rigid base, K's go to zero and first row can be written

$$\left[\omega_{fix}^2 - \omega^2 I\right]\{p\} = \omega^2 M_{ps}\{s\}$$

- Now one can back out the rigid body s dof required to get the appropriate p dof which approximate the elastic motion (strain)

$$\{x\} \approx \Phi\{q\} = \Phi T\begin{Bmatrix}p\\ s\end{Bmatrix} = \Phi T\begin{bmatrix}\left[\omega_{fix}^2 - \omega^2 I\right]^{-1}\omega^2 M_{ps}\\ I\end{bmatrix}\{s\}$$

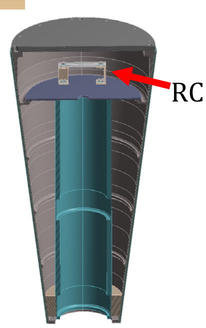
6 s dof with a rigid fixture

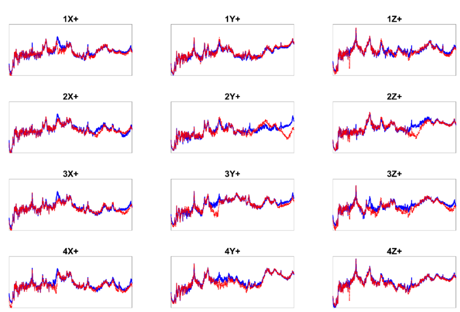


5

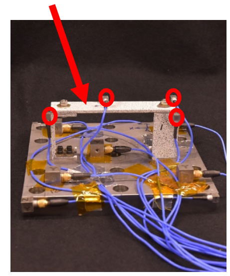
Resulting Craig Bampton modal model with the Removable Component (RC) from the BARC round robin structure

- Blue RC data in system env / Red CB model simulation





RC with 4 triax

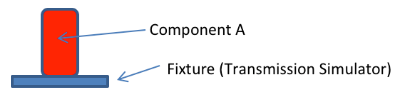


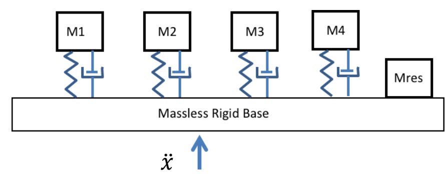
Modal test structure

6

Effective Mass Modal Model Description

- Effective mass models simulate response of a component attached to a rigid fixture – This fits in nicely with the transmission simulator concept
- The modal model represents the response in only one direction to the acceleration of the base, which is the input (not a force but an acceleration)
- The masses, springs and dampers provide the fixed base modal frequencies and damping





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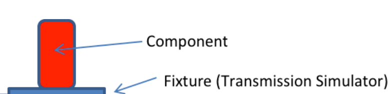
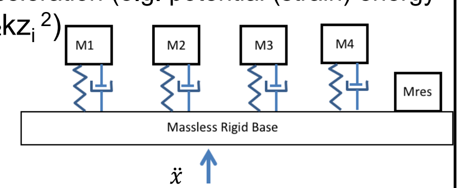
7

Traditional Effective Mass Modal Model Uses

- Payload loads analysis – Modes with high effective mass are the major modes that put large loads back into the base (the bus carrying the payload)
  - kz_i where z_i is the displacement between mass i and the base gives physical force
  - used to validate the “important” modes in FE models
  - In earthquake engineering, effective mass is the part of the mass responding to quake
- Margin analysis – Tests to failure can characterize the margin of the component above the qualification level through the energy metrics that come from a particular base input acceleration (e.g. potential (strain) energy in a mode can be calculated as $\frac{1}{2}kz_i^{\,2}$)





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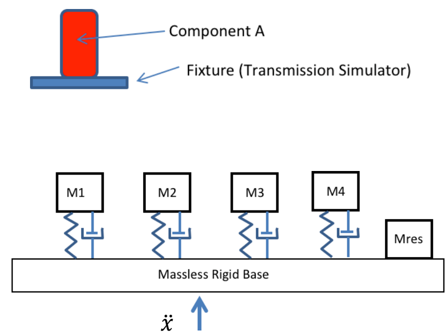
8

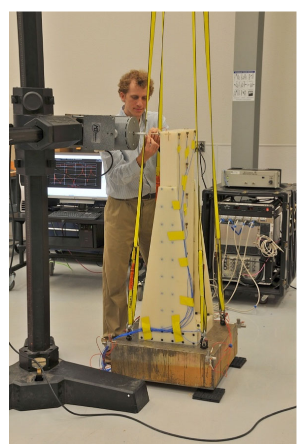
Effective Mass Modal Model Features

- The residual mass is the sum of all modal masses above the frequency of interest
- The effective masses sum to the total mass of the component, therefore scaling is not arbitrary, but related to physical force and displacements
- Useful for describing vibration or shock tests performed in one axis at a time



Proof of Concept Example 1 – Payload Example

- 72 kg nylon beam tested two ways
- Note beefy fixture – the first elastic mode of the fixture is over 1300 Hz
- Floppy fixtures (with lots of modes in the desired bandwidth) may require an excessively large number of modes in the bandwidth to give reasonable fixed base estimates



$$\left[\omega^2{}_{free} + j2\omega\omega_{free}\zeta_{free} - \omega^2 I\right]\bar{q} = 0$$

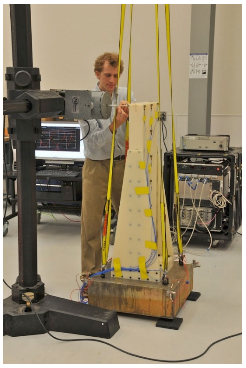
$$\Psi^{+}\,\Phi_b\bar{q} = s = 0$$

$$\bar{q} = L_{fix}\Gamma\bar{p}$$

Proof of Concept Example 1 – Payload Example

- Effective mass from experiment compared to validated FEM

Test Frequency (Hz)	Effective Mass from Test	Effective Mass from FE Model	Difference as % of Total Test Article Mass
38.94	0.415	0.448	3.3
163.5	0.187	0.189	0.2
396.3	0.088	0.085	-0.4
706.4	0.037	0.047	1.0
859.2	0.004	0.000	-0.4
1034.7	0.011	0.000	-1.1
1048	0.003	0.023	2.0
1190.7	0.000	0.000	0.0
1316.1	0.027	0.004	-2.3
1344.3	0.020	0.004	-1.6



Proof of Concept Example 2 – Component Example

- Circuit board on fixture
- Proof of concept example is on right – FE model was used as point of reference

Proof of Concept Example 2 – Component Example

- Circuit board on fixture
- Proof of concept example is on right – FE model was used as point of reference

Test Frequency (Hz)	Effective Mass from Test	Effective Mass from FE Model	Difference as % of Total Mass of Test Article
339.4	0.815	0.816-0.831	-0.1 to -1.6
1081.4	0.069	0.058-0.062	-1.1 to -0.7
2705	0.040	0.0041-0.0043	-0.01 to -0.03

13

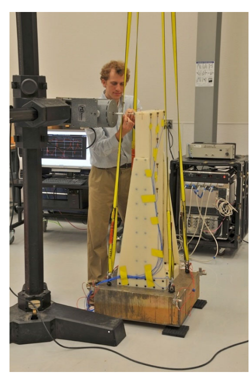
Effective mass model can be extracted from CB Modal Model

- Perform a modal test, calculate fixture mass properties and component mass properties

$$\left[\omega^2{}_{free} + j2\omega\omega_{free}\zeta_{free} - \omega^2 I\right]\bar{q} = 0$$

$$T = \begin{bmatrix} L_{fix}\Gamma & \Phi_b^{+}\Psi \end{bmatrix}$$

$$\left[\begin{bmatrix} \omega_{fix}^2 & K_{ps} \\ K_{ps}^T & K_{ss} \end{bmatrix} - \omega^2 \begin{bmatrix} I & M_{ps} \\ M_{ps}^T & M_{ss} \end{bmatrix}\right] \begin{Bmatrix} \bar{p} \\ \bar{s} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \bar{F}_s \end{Bmatrix}$$



Note: scale Ψ to 1s and 0s for appropriate effective mass scaling.

14

Theory using the transmission simulator for more general form from which effective mass can be extracted

$$\left[\begin{bmatrix}\omega_{fix}^2 & K_{ps}\\ K_{ps}^T & K_{ss}\end{bmatrix} - \omega^2\begin{bmatrix} I & M_{ps}\\ M_{ps}^T & M_{ss}\end{bmatrix}\right]\begin{Bmatrix}\bar{p}\\ \bar{s}\end{Bmatrix} = \begin{Bmatrix}0\\ \bar{F}_s\end{Bmatrix}$$

0's for rigid body modes

From first row partition

$$(\omega_{fix-i}^2 - \omega^2)p_i = \omega^2 M_{ps-ij}s_j$$

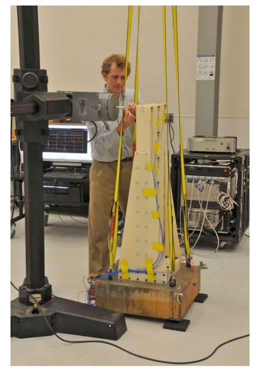
$$p_i = \frac{\omega^2 M_{ps-ij}}{(\omega_{fix-i}^2 - \omega^2)}s_j$$

From second row partition

$$-\omega^2\sum_i M_{ps-ij}\frac{\omega^2 M_{ps-ij}}{(\omega_{fix-i}^2-\omega^2)}s_j - \omega^2 M_{ss-j}s_j = F_j$$

For very large ω $\quad (-\sum_i {M_{ps-ij}}^2 + M_{ss-j})\ddot{s}_j = F_j$

$$m_{eff-ij} = M_{ps-ij}^2$$



Effective Mass Parameters

- Set effective mass = $m_{eff-ij} = M_{ps-ij}^2$
- Set effective stiffness = $\omega_{fix-i}^2 M_{ps-ij}^2$
- Mres is all the mass that is not considered in the spring mass modes